Definition 1. *An Auerbach system in a Banach space X is a system $(u_\alpha, g_\alpha)_{\alpha<\kappa}$ such that*

- $u_\alpha \in X$, $\|u_\alpha\| = 1$
- $g_\alpha \in X^*$, $\|g_\alpha\| = 1$
- $g_\alpha(u_\beta) = \delta_{\alpha,\beta}$ *for* $\alpha, \beta < \kappa$.

A remarkable result of Hájek, Kania and Russo states that under CH there is an equivalent renorming of $c_0(\omega_1)$ without uncountable Auerbach systems ('a counterpart to Kunen's line within the class of WLD Banach spaces').

It is natural to ask whether this result holds in ZFC. Struggling to patrially answer this question we enter the world of determinants, analytic functions, Fubini products of ideals and strongly Lusin sets.